Ec453 Theories of Growth and Development
Homework on Intertemporal Optimization

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(I) Consider the standard Ramsey Cass-Koopmans model of growth.

Assume that aggregate output *Y=F(K, L)* is given by the Cobb-Douglas function, with

$$F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}$$

Suppose that agents in this economy have the following representative intertemporal felicity function:

$$U = \int_{t=0}^{\infty} e^{-(\rho - n)t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt$$

where ρ>0 is the subjective discount rate. Capital labor ratio evolves over time with: $\dot{k} = f(k) - \delta k - nk - c$, where d is capital depreciation rate, *n* is population growth rate and *c* is per capita consumption.

Assume that the economy is closed to foreign trade. In this economy, the consumer tries to choose an optimal consumption profile by maximizing the discounted utility derived from the stream of future consumption subject to the budget constraint.

$$\dot{a} = w + ra - na - c$$

where *a* is assets per person, and *r* is the rate of return rate on assets.

1. Using the *Hamiltonian* of this system which maximizes per capita consumption subject to the capital accumulation constraint, find the long run equilibrium rate of growth of per capita consumption for this economy.
2. Solve the producer's problem and derive the necessary conditions for the neoclassical steady sate.
3. Draw the transition to the steady state path within a phase diagram over the space of consumption per labor and capital per labor.
4. Now suppose that the government decides to implement a tax on asset incomes. Thus the budget constraint of the agent becomes:

$$\dot{a} = w + (1-t)ra - na - c$$

where *t* is the tax rate on asset incomes. Re-work your solution to the optimal consumption path. How are the first order conditions towards the steady sate affected?

5. Show clearly the net effects of the tax on the new steady state equilibrium. Explain your result briefly.
6. Observe that the consumption maximizing (the golden rule) k lies to the right of the steady state k. Prove this analytically. Discuss reasons why this might be so.

(II) As we have seen in class, neoclassical growth theory sees the per capita differences across countries as a result of differences in saving rates and in gaps in technology. We have seen that, given a Cobb-Douglas representation of national output, per capita output under steady state can be expressed as:

$$\frac{Y}{N} = s^{\alpha/1-\alpha} A$$

Where s is savings as a ratio to the GDP, and A is an index depicting the state of technology. There is a mythological belief within the international academic community that the capital share in output, a, is roughly 0.3; so simply accept this number as given. Given the above formula, and assuming that technology is readily available for all countries (so all countries have identical A), use your data (from the Summer-Heston Penn tables) on per capita levels and saving rates (savings as a ratio to the GDP) for the selected countries above.

How does the above neoclassical formula fare in predicting and explaining your data in terms of per capita income differences between the US and the remaining countries? How can you improve upon the predictive powers of the above formula?