

Ec 453 Theories of Growth and Development I
Fall 2014, Homework On Neoclassical Growth-II

Prof. Erinc Yeldan

1. In the country of Douglasia, output is produced along the production function?

$$Q = K^\alpha L^{(1-\alpha)}$$

where Q is output, K and L are capital and labor respectively.

- (a) Write the output per labor function for Douglasia. Find an expression of the rental rate of capital (the profit rate) and the wage rate, and the factor shares in terms of the per capita production function parameters.
 - (b) Suppose that the growth rate of labor force is zero, and the rate of depreciation of capital is 2%. The Douglasians are currently saving 20% of their national output for investment purposes. Further suppose that α is a known parameter estimated to be 0.5. Calculate the steady state capital-labor ratio for Douglasia. Sketch a graph.
 - (c) Analytically derive a rule for the per capita maximizing consumption rate of savings for Douglasia. Find the levels of per capita consumption, savings, and the capital labor ratio under the new steady state.
 - (d) Calculate the *Golden Age* of this system, as Thompson referred to it as the *per capita profit maximizing* levels of per capita consumption, savings, and the capital labor ratio under the new steady state, and compare them with the findings in (c), that is the *Golden Rule of Consumption*.
 - (e) Now suppose that a group of young graduates of Bilkent Economics has come with a non-refutable finding that in Douglasia the share of capital, i.e. α , is actually 1.00. Calculate the new steady state and draw the “transitional dynamics” of the economy under this new specification.
2. Consider the neoclassical (Solow-Rebelo) model of *almost-endogenous* growth with the following technological set up: Single output Y is produced by labor, L , and capital, K . The production technology is:

$$Y = AK + BK^\alpha L^{(1-\alpha)}$$

where K and L denote capital and labor inputs, respectively. The parameters A and B show the technical productivity levels.

There is no technological progress otherwise, and growth of labor is also assumed to be zero. Capital depreciates at the rate δ . A fraction s of output is saved and invested for capital accumulation. For simplicity assume that stock of labor, L , is assumed to remain constant at unity.

- (a) Show the transition path of the capital labor ratio for this economy towards the steady state.

- (b) Is there a unique steady state for this economy? Discuss why, or why not? What can you say about the properties of the rate of return to capital per labor, r ?
3. One of the most important policy conclusions of the Solowian neoclassical growth is that the per capita income levels of all nations will converge in the long run.
- (a) Discuss the conceptual hypotheses behind this prognostication. Under what conditions can we obtain convergence as stated in the neoclassical framework?
 - (b) In order to test the empirical validity of this assertion, go to the Temple's Penn World Tables at https://pwt.sas.upenn.edu/php_site/pwt_index.php and choose any 20 countries that you find interesting. Draw upon the data on per capita income levels of these 20 countries and that of the USA, the world's leading economy over the post-War era, 1950-2000. Map your data on a graph where on the vertical axis you depict the average annual rate of growth of these countries, and on the horizontal axis depict the level of per capita income back in 1950. If the theory were correct, then you should have a downward sloping fitted line across your observations. Test this analytically and discuss whether the convergence hypothesis holds for your sample, or not. Discuss the reasons of your findings.