Chapter III. The Simple Dynamic CGE Model of a Small Open Economy

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III-1. Introduction

In this chapter, the analytical model developed in Section II.4 and II.5 of the previous chapter is fit to country level data at a level of aggregation equivalent to the analytical model. A challenge is to organize the economy-wide data into a logically consistent structure and to extend the analytical model to account for various intricacies in the data and the "real world" we wish to model.

The construction of a consistent data set itself can be a tedious exercise and, needless to say, several problems are encountered. The most important among these, are the problems that arise from the aggregation of economic objects. In the abstract treatment of commodities, by aggregating seemingly alike categories of goods (along with factors and even households) one implicitly assumes homogeneity within each category. In the realm of numerical modeling of course, one necessarily has to deal with aggregation of heterogeneous objects. Whatever the principle might be, the aggregation procedure necessarily involves the putting together in one category intrinsically diverse objects. These and several other problems common to the specification of empirical models are considered in this chapter.

A special case arises with commodity aggregates that are traded internationally. Even at moderate levels of aggregation, the phenomenon of two "way trade" is observed. That is, for a given sector, both import and export activities are often observed to occur simultaneously. This observation clearly runs counter to the theoretical model structure in which a positive or negative excess demand for a homogenous good determines whether a sector is an exporting or importing sector, but cannot be both. Effectively, the home and foreign good of the same category are in fact imperfect substitutes.

Another example is the *heterogeneity* among factor endowments that are otherwise in the same category. Labor skill types, for instance, their corresponding markets may be segmented or loosely linked. This diversity may necessitate the taking into account the various layers of diverse characteristics and behavior patterns in labor markets. Related to the heterogeneous structure of factors is the very indigenous nature of the technology. Once account is given to characterization the naturally diverse sectors, such as primary agriculture and manufacturing industries, one has to deal with non-homogenous production technologies, and the realistic portrayal of whether they are vertically or horizontally integrated.

In addition to these concerns is the problem of only attempting to model the real side of an economy. It is known that modeling the real component of an economy ignores the affects of monetary transactions on business cycles and the feedback to the real component of the economy. Thus care must be taken in the specification of the trade (im)balance, inflationary dynamics, determination of exchange rates and the behavior of other financial variables.

Naturally, there exists no "universal" model to deal with all aspects of the "real world" economies simultaneously in one thrust of theoretical consistency. A one-to-one correspondence to the actual economic behavior is a negation of the purpose of model building in the first place. Clearly, a "powerful" model is *not* the one, which narrates as much detail about real life as possible; but instead, one, which is sharply, focuses on a well-stated policy problem that captures as much of the *related* detail as possible with the weakest set of assumptions. Thus, one necessarily has to be selective in emphasizing the relevant aspects of the "economic machine" over which the focus of policy analysis is to be directed.

Our purpose is to build the links between the "focused" reality and the analytical structure designed to understand it in the first place. For this, we utilize a two-sector, two-factor specification of the dynamic small open economy model presented in Sections II-4 and II-5 in the previous chapter. In the next section, we focus on how to generate and organize the data into a Social Accounting Matrix (SAM). This is an important step in the numerical implementation of a dynamic computable general equilibrium (DCGE) model. The next step is to calculate the key parameters of the analytical model from the data displayed in the SAM. This is accomplished in Section III.3. In this section we also extend the simple model to account for intra-industry trade. In Section III.4, the equations for numerically solving the model are laid out in a way that is generic to virtually all of the DCGE models presented in this book. The simple model is then solved and shown to reproduce in the SAM exactly. Other solutions are also derived to illustrate many of the models features discussed in Chapter II. :

III-2. Organization of the Data for the General Equilibrium System: The Social Accounting Matrix

Organization of the data for any applied modeling work is an essential component of model building efforts. The applied general equilibrium methodology has, in the course of its development, generated a need for the reconciliation of a set of data on national income, production and public accounts and identities in a consistent framework. Drawing heavily on the earlier influential work of Richard Stone, the founder of the United Nations System of Accounts (SNA), such a framework has evolved, and referred to as a Social Accounting Matrix (SAM).[1]

Major data used to construct a country's SAM for the purpose of single country analysis include the data about national income, production, consumption and public accounts, which can be found in the national income accounting statistics and the so-called input-output tables. For the purpose of multi country modeling analysis, export and import data by source-of-origin are also required and can be found from the countries foreign trade and customs statistics.

A country SAM is built around the principle of "accounting identities". The underlying premise is that the data presented gives a snap shot of all the major flows of income sources and expenditure categories of an economy in a specific time period, usually, one year. As such, it portrays a global picture of all the major economic transactions among the various agents of an economy for an interval of time, usually a year.

The SAM is organized around the general principle that the rows identify a country's "income" flows or "receipts", and the columns reflect the "expenditure" flows (or uses of the receipts) for each account. This principle, together with the accounting identities, imposes the overall constraint that the sum of each row (receipts) must be equal to the corresponding column sum (expenditures). In other words, receipts and expenditures of any transaction activity must balance.

At the outset, there does not exist a universal standard guideline in the design of a SAM. This is due to the fact that its accounts necessarily have to be tailored to the specifics of the problem at hand. The point here is that the specific design of the SAM follows the intrinsic structure of the theoretical model to which a SAM serves as an organized, consistent data set. If the current focus of the model is mostly on say, structural adjustment and trade balance, then the accompanying SAM highlights the production and the expenditure accounts; or in contrast, if the model emphasizes income distribution issues, then the SAM should naturally provide a more detailed treatment of the factor income generation accounts.

Next, we introduce the major underlying activities of an economy in the SAM framework and discuss the accounting identities with which they are linked in the Walrasian

[1] SAM references here...

system. We start from a highly aggregated SAM corresponding to the economic agents and the activities of the theoretical model provided in the previous chapter. Then, we expand this simple SAM to account for multi-sectoral production and other expenditure activities. Finally, we introduce policy activities into the SAM.

III.2.1 The SAM of a one sector open economy

Consider a highly aggregate open economy as we described in the Section II-3. The value of the single output produced in a specific year which amounts to Y (value of the *gross national product*) is sold in the domestic market and exported. The value of gross national product equals national income, which is spent on purchasing consumption goods and savings. Savings are used to invest or lend to the foreign country (of course, a negative lending is to be interpreted as borrowing from abroad). Purchasing domestic goods satisfies the value of total consumption and investment. The difference between investment and domestic savings corresponds to the imbalance in trade, i.e., a trade deficit or surplus.

We organize these activities into four components and portray them in a tabular form in Table III-1. In this simple structure, the value of output generated from "production activities" is sold in the domestic market, DC, and exported, E. Thus along the first row, the "revenues" of the production activities are DC from the *domestic market activities* and export revenues, E, from the *trade activities*, i.e.,

$$Y = DC + E$$

The "revenue", Y, then become the income of the private *agent* along the first column. The *agent* allocates income to consumption C and savings, S, i.e.,

$$Y = C + S.$$

The column of market activities $(DC+M)$ captures the information about the source of commodities, i.e., from where the commodities originated (in the National Income Statistics this magnitude is referred to as *market absorption*); while the row of market activities $(C+I)$ displays the different uses of the aggregate commodities, in which C represents the value of goods consumed and I is the value of goods used to add to capital stock. The row of market activities is called a receipt for the *domestic market activities.* Thus, we have

$$C + I = DC + M.$$

Expenditure on the goods for the purpose of investment does not necessarily equal domestic savings. The difference between I and S is met by borrowing from abroad. This magnitude corresponds to the trade deficit, $M - E$, and is registered in the row of capital accumulation activities and the column of trade activities, so that the fundamental accounting identity,

$$I = S + TDEF$$

Is satisfied.

As can be observed from this simple stylized SAM, for each activity, the corresponding expenditures (column sum) and receipts (row sum) must be equal. Thus, one can phrase the well-celebrated corollary of the Walras law in the SAM framework: “if all, but one, of the SAM activities are in balance, the remaining one must also be in balance”. This accounting identity highlights the basic underlying relationship between the SAMs and the Walrasian (neoclassical) general equilibrium systems.

III.2.2 The SAM of a multi-sector open economy

Building on the above structure, we expand the simple SAM so that in later sections we can account for a multi-sector, multi-factor economy. In so doing, we also implement a richer description of the production activities. In Table III-2, we portray the accounting system of a two-sector open economy with two primary factors of production, capital and labor, and intermediates. Note that we now add a set of columns/rows for the production factors, and also a new activity, which we label *policy activities.* Among “policy activities” we have in mind policy instruments such as production taxes and tariffs. We could have also chosen to separate government from private households as an independent account entry. However, in this introductory SAM framework we prefer to follow the typical treatment which is simply to transfer tax revenues in lump sum back to households.

In the SAM portrayed in Table III-2, we now distinguish two production activities: sector-A and sector-N. We carry the same distinction over to the domestic market activities, i.e., commodity-A and commodity-N. The “policy activities” account implement production and import taxes, and transfer the collected tax revenues to the private households.

We focus on the new account entries that do not appear in the previous SAM. The first of these new accounts is the *activities* between firms captured by the four cells linking production and market activities. For example, reading the first “Activities” column -- “Activity-A”, we see that the sector-A purchases commodity-A and commodity-B as intermediate inputs. Similar activities are observed for sector-N under the second “Activities” column.

The second new activity is the purchases of factor services by production sectors. The cells linking activities and factors capture this. With these additions, household incomes can now be distinguished by sources, i.e., as incomes originating from the labor and capital services.

The government taxes on producers are captured by the cells linking "activities" and "government policies" and appear as a cost of production. Thus, total production costs are intermediate input costs plus primary input costs and production taxes. These are observed by the row sum for the first two columns of activities.

With the presumption of a constant returns to scale technology, total production costs equal total production revenue. We can observe this equality by comparing the column of activities (costs of production) with row activities, i.e., the production revenue captured by the domestic supply of *DC* and export, *E*.

Commodities that can be allocated to various purposes are still observed in the row titled "commodities"; while the sources of commodities absorbed are in the column titled "commodities". Here, a new activity is the *government's import tariffs* in the intersection cell of "government policy" and "commodities". As all numbers reported in a SAM are in value terms, and presumably, observed outcomes of domestic ~~market~~ economic activities, the value of imported goods faced by the domestic consumers are in fact different from the value of these goods at border prices. Hence, the cells linking "trade" and "commodities" depict imported goods at border prices. This number plus tariffs is the domestic value that consumers have to pay for the imported goods.

Without the government account, tariff revenues, in fact, are transferred to households, which are captured by the cells linking "households" and "government policy". Aggregate household income (which can be read from the row of "households") now equals income from factor services plus transfers of the total tax revenues. Similar to the previous SAM, household incomes are spent on consumption goods and savings. Also, the gap between household investment and savings has to be filled by foreign borrowing that is dietetically equal to the trade gap between imports and exports (both are valued at the border prices).

The calculation of gross national product or national income now can be made precisely. Conceptually GNP can be valued either at *factor costs*, or at *market prices.* The GNP valued at factor cost is the sum of value added; i.e., the sum of wage payments to labor and returns to capital. When various production taxes and tariffs are added to this magnitude we arrive at the value of *GNP at market prices.* In terms of the conceptual framework described in Chapter II, the GNP function outlined previously corresponds to the GNP measured at factor costs in the current SAM; that is, wage payments plus returns of capital.

A numerical example of the SAM accounts identified thus far is presented in Table III-3. The data appearing in Table III-3 come from an aggregation of the 1990 macro balances of the Turkish economy (Köse and Yeldan, 1996). Compared with the discussion above, readers should easily follow the relationships among the values in this table. We thus skip such discussion of these particular values.

We now turn to the issues of numerical implementation (*calibration*) of the given SAM data to the algebraic structure of our simple dynamic general equilibrium model.

III-3. Implementation of the SAM Data: Calibration of the Algebraic System of Equations

By now, the fundamental principle of the SAM-based data is clear: they portray the flow of macroeconomic identities in a consistent system of accounts. They provide a comprehensive overview of the major transactions of a market economy and can reconcile both the micro-agents, such as the factors, consumers, and the firms, with the macro-units, such as savers, investors and the public sector. In this respect, the SAM is an indispensable tool for organization data and classifying the structural flows of the economy.

However, it has to be noted that the SAM alone does not presume any conceptual apparatus. That is, it does not specify or depict any behavioral or institutional characteristics of an economy. For this reason, the SAM should not be considered as a tool of policy analysis. Instead, it should be viewed as an essential component to organizing data for the next stage of analysis. Underlying each account in the SAM structure, a component of a Walrasian economy can be envisioned. To express it in Thorbecke's (1985: 207) words: "If the SAM is to be used for policy rather than purely for diagnostic purposes, it *has to be coupled with a conceptual framework that contains the behavioral and technical relationships* among variables within and among sets of accounts and modules. In other words, the SAM as a data framework is a large-scale identity which, *to come alive, should be linked to a model* of the casual relationships among variables" (emphasis added).

Thus, in this sub-section our purpose will be to provide the theoretical basis of Chapter II to our stylized SAM introduced in Table III-3. Through this exercise we will be able to bring life to both the numerical values of the SAM and also to the abstract theory. To satisfy the necessary links between these two structures, however, further assumptions are required.

The first and the most important assumption typically imposed on the data is that a market clearing - equilibrium process, generated it[2]. If the SAM is constructed for a *static* general equilibrium analysis, the data reflected in the SAM is the outcome of an economy in equilibrium. The existence of equilibrium not only implies that the data organized in the SAM are balanced, but also that they are derived from the outcomes satisfying the rationality assumption for each agent in the economy. In addition, when the SAM is used for the *dynamic* general equilibrium analysis, the data have to also represent the economy in its *long-run equilibrium*, i.e., an economy in steady state equilibrium.

Specification of the SAM s data as the annual outcome of an economy in long-term equilibrium allows us, on the one hand, to refer to the data as a benchmark against which alternative policy scenarios can be contrasted. Another important purpose for the equilibrium assumption is that it allows us to calibrate the many of the key parameters of the model.

For the purpose of parametric calibration, we also need to assume specific functional forms for the primitives of the empirical model, including production technologies and consumer preferences. Another important feature in this regard is whether *constant returns to scale* are required or whether functions need to satisfy, e.g., the Inada conditions.

It should come as no surprise that it is impossible to generate all parameters endogenously from a single data set such as SAM. Recall that the SAM portrays data only on the *flow* variables, leaving the *stock* values to be obtained from outside sources. There will be a further need to obtain estimates of various parameters such as the elasticity of factor substitution from sources other than the SAM.

Keeping these assumptions and limitations in mind, the calibration allows us then to regard the SAM data set *as a one-period snap shot of an economy in equilibrium*. Calibrating the parameters of a model to this equilibrium provides a *base-run* solution of the numerical model that exactly reproduces the data, i.e., the one period snap shot. Thus, this snap shot can serve as a base to which other numerical experiments of the model can be compared.

We will proceed below with a discussion of the underlying reduced form equations and their calibration in “blocks” of sub-models.

III-3-1. The Price System and Foreign Trade under Product Differentiation

Since the data organized in the SAM are in *value* terms they entail products of real quantities and corresponding prices. To conduct any consistent analysis, we have to separate price from quantity for each variable since individual agents are usually presumed to choose quantities, treating prices as given. However, without additional information either about price or quantity of each variable, it is impossible to separate price from quantity for all the variables. This problem is easily circumvented. It is both convenient and theoretically consistent to regard the "nominal" values of the data as the "real" values of quantities for the base year, i.e., treating prices of most commodities as one[3]. This treatment is theoretically consistent because our exclusive focus is on the *real* economy in which monetary terms are absent. Hence, in what follows, we will treat most of the SAM data as reflecting the *quantity* values measured in base-year prices with a price index of unity for each.

The open economy specification presumes in principle that world prices prevail in the domestic markets. However, due in part to the aggregation of the production units, the hypothesis of homogeneity, i.e., substitutability among the same category of home and foreign produced goods that are traded, can lead to an unrealistic situation where some of the domestic sectors have to be closed, while some others eventually dominate the whole economy. Obviously, this runs counter to the data which suggests that many sectors both export and import within the same good category simultaneously. That is, two-way trade is common, and tends to increase at higher levels of aggregation. Applied multi-sectoral models necessarily involve a fair amount of aggregation of sectoral activities and, at such levels of aggregation, the homogeneity assumption is not viable.

An approach to this problem was proposed in a 1969 paper by Armington which distinguishes commodities not only by their kind --e.g. machinery, chemicals, ...-- but also by their place of production. In the Armingtonian composite commodity system, not only is each good different from any other good, but also the country of origin of supply differentiates each good. Following this specification, domestically produced goods and imports in the same good category, e.g., agricultural goods are assumed to be imperfect substitutes. As such, the Armingtonian commodity and the associated price system became an integral component of modeling general equilibrium.

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[2] Of course, this does not preclude the use of a SAM to depict and model a centrally planned economy.

In general, *calibration* of any functional parameter has to assume that the underlying solution in fact satisfies the principle of optimality (the foundation of an equilibrium). Given the CES and quasi-concave assumptions, the first order conditions are sufficient to guarantee that the respective solutions are indeed optimal. Hence, it is common to employ the first order conditions to conduct the calculation of parameters for major equations.

We first discuss the calibration of the parameters in the celebrated *Armington commodity system.* The composite Armington good, *CC*, is formulated From a CES function in which the domestic commodity, *DC,* and the imported foreign good, *M* are served as "inputs":

$$CC = AC\left[\beta M^{-\upsilon} + (1-\beta)DC^{-\upsilon}\right]^{-1/\upsilon} \tag{3.1}$$

The elasticity of substitution of the CES function, σm, and the parameter n are related through, $\sigma m = \frac{1}{1+\nu}$; while the parameter, β, (the share parameter) and AC (the shift parameter) are needed to be calibrated from the SAM.

As all agents in the economy are assumed to choose their respective consumption levels to minimize total purchase costs subject to the CES composite commodity "technology", this problem is akin to that of choosing inputs to minimize production costs subject to a given technology. Accordingly, *M* and *DC* are like "inputs" producing the composite, *CC*. Hence. The first order conditions of this problem can be stated as follows:

$$\frac{M}{DC} = \left[\frac{PD}{PW(1+tm)}\right]^{\sigma m}\left[\frac{\beta}{1-\beta}\right]^{\sigma m} \tag{3.2}$$

In Eq. (3.2) and all following equations used to calculate parameters, a variable with bar means that it is a number obtained from the SAM. Thus, the share papermaker, β, and then the shift parameter, *AC*, can be calculated from this equation by the following processes: from Equation (3.2) we obtain,

$$\frac{\beta}{1-\beta} = BETA = \left(\frac{M}{DC}\right)^{1+\sigma m}\left(\frac{PW(1+tm)}{PD}\right) \tag{3.3}$$

Thus,

$$\beta = \frac{BETA}{1+BETA}. \tag{3.4}$$

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[3] If a country s sectoral labor supply data are available, then it is easy to incorporate these data into a SAM. By so doing, wage rates may differ across sectors and hence are not necessary equality to unity.

Using the calibrated value of b, the shift parameter can then be found by using:

$$AC = \frac{CC}{\left[\beta M^{-\upsilon} + (1-\beta)DC^{-\upsilon}\right]^{-1/\upsilon}} \tag{3.5}$$

Recall that all prices appeared in the equations is assumed to be one, and that *DC* and *M* are observed from the data in the SAM.

The notion of imperfect substitutability among goods in the same category also carries over to the production side between the production for domestic sales, *DC*, and for exports, *E*. Just like the demanders, the producer is considered to solve an optimization problem of allocating output, *XS*, to both markets, *DC* and *E*, so as to maximize revenues subject to the transformation technology.

$$XS = AT\left[\eta E^{\tau} + (1-\eta)DC^{\tau}\right]^{1/\tau} \tag{3.6}$$

where the transformation elasticity, $\sigma e = \frac{1}{\tau - 1}$, captures the relative smoothness of how the produced output, *XS*, can be marketed as domestic sales, *DC*, or exported, *E*. To calculate the share (h) and the shift (*AT*) parameters in the CET function we make use of the first order conditions as follows: Starting from the optimal exports to domestic sales ratio,

$$\frac{E}{DC} = \left[\frac{PW}{DC}\right]^{\sigma e}\left[\frac{1-\eta}{\eta}\right]^{\sigma e} \tag{3.7}$$

We obtain the value of h directly:

$$\eta = \frac{1}{1 + \left(\frac{PD}{PW}\right)\left(\frac{E}{DC}\right)^{\sigma e - 1}} \tag{3.8}$$

Finally, the shift parameter can be calibrated given (3.6) and (3.8):

$$AT = \frac{XS}{\left[\gamma E^{\tau} + (1-\gamma)DC^{\tau}\right]^{1/\tau}} \tag{3.9}$$

and similar as in Armington calibration, *DC* and *E* are the data from the SAM by assuming the unit price for E.

III-3-2. Structural Parameters of the Production Function

We posit a Cobb-Douglas production function to express the relationship between primary inputs, labor and capital, and the outputs, and a set of fixed input-output coefficients, which reflect the relationship between intermediate inputs, and the outputs. In this manner, denoting labor and capital, respectively, L and K, the intermediate inputs by Vij, and the Leontieff input/output coefficients by aij identify the relevant production technology identified by the following equation:

$$XS_i = \min\left\{\frac{V_{1i}}{a_{1i}}, \frac{V_{2i}}{a_{2i}}; AX_i \cdot L_i^{\alpha_i} \ K_i^{(1-\alpha_i)}\right\} \tag{3.10}$$

To calibrate these functional forms to the SAM data, we again utilize the optimality conditions on factor use. The producers demand labor up to the point where its marginal product is equated to the wage rate. Thus, using the first order condition for profit maximization, we infer the value of a directly using:

$$\alpha_i = \frac{Sectoral\ Wages}{Sectoral Value\ Added} \tag{3.11}$$

for each sector. Observe that by normalizing wl = 1.0 as before, the total wage bill, can be read from the SAM data, under the cell "Wages". We postpone the calibration of capital good demand to section III-3-4, where the dynamics of the model are introduced.

Finally, the input-output coefficients are calibrated from the SAM cells on sectoral intermediate input use:

$$a_{ij} = \frac{INTERMEDIATE\ flow_{ij}}{Value\ of\ Output\ Supply_j}. \tag{3.12}$$

III-3-3. Structural Parameters of the Demand System

In Chapter II, we mention that the household demand for individual commodity can be derived separately given intertemporal allocation of income on consumption expenditure for each time period. By assuming the technology to generate an aggregate consumption good from each individual good is of Cobb Douglas form, the share parameters of this function, denoted $cles_i$, can be easily calculated from the data in the SAM by simply dividing the expenditure on each good by total consumption expenditures, i.e.,

$$cles_i = \frac{Sectoral\ Consumption_i}{Total\ Consumption\ Expenses} \tag{3.13}$$

Notice that this procedure is the same as using the first order condition for the second level of household problem.

Similarly, we assume that the technology to produce an increment of new capital is of Cobb Douglas form using the two final goods as inputs. Thus, the data in SAM provide enough information to calculate the share parameter for each of the inputs in the capital production function. Using the SAM data for sectoral investment demand expenditures, we obtain the sectoral investment share parameters, $cles_i$, directly:

$$iles_i = \frac{Sectoral\ Investment\ Demand_i}{Total\ Investment\ Expenditures} \qquad (3.14)$$

III-3-4. Specification of the Benchmark Steady State

The algebraic structure of the intertemporal segment of the model is based on the neoclassical growth theory, the basic principles of which are laid in the previous chapter. For specification of the variables and the structural parameters governing the intertemporal system, we observe that the SAM apparatus is of little use. This is to be expected, since SAMs are mostly static artifacts, portraying exclusively snapshots of the macro accounts at one instant in time. They generically display *flow* data, which are not suitable for characterizing the initial position of the stock variables, such as capital stocks and debt/asset holdings.

In fact, among this set the most difficult task pertains to the information about the initial stock values of these variables. The aggregate stock value and the net foreign debt/asset position of the economy provide crucial information about the disposal of domestic savings and the evolution of the cumulative factor --capital-- over time. Thus, their consistent specification within the discipline of intertemporal general equilibrium is an important and no trivial task.

Furthermore, certain parameters such as the subjective discount rate (rate of time preference, ρ) and the elasticity of intertemporal substitution are structural parameters of the analytical model. As such, they tend to have no clear and direct counterparts in the real world data and their estimation from a single SAM is not feasible. So for such variables, reliance on other studies seems to be an only viable option.

Parameters such as the interest rate and the depreciation rate, on the other hand, can be extracted from the outside data sources. However, it has to be borne in mind that the intertemporal system of equations are governed by their internal logic of theoretical consistency and one cannot arbitrarily make use of ad hoc outside values on such parameters. Simple incorporation of such data from other research results and/or arbitrary approximations could easily lead to disequilibria and inconsistencies between the existing SAM data and the analytical logic of the model.

Therefore, a convenient assumption that also proves to be analytically consistent is to regard the existing SAM data as reflecting *long run steady state.* In other words, we interpret the flow data portrayed in the SAM, not as being derived from a *static* picture, but as part of a dynamic system which has achieved its *intertemporal, steady state equilibrium.* This assumption not only facilitates the derivation of many important variables of the model in line with the existing data, but also enables a direct and consistent link between the SAM construct and the analytical results derived from the economic theory. Furthermore, this interpretation, on the one hand, minimizes the risk of making ad hoc and/or arbitrary assumptions regarding the evolution of the modeled economy; and, on the other, provides a blueprint on how to parameterize the dynamics of steady state equilibrium.

Based on this interpretation, we first discuss the empirical specification of the consumer's problem. Given the setting outlined in chapter II, we envisage the consumer as an infinitely-lived to maximize an intertemporal utility function. As noted there, the consumer's utility maximization problem is a two-level activity: at the *first* level, we view the infinitely-lived households to consume home produced and imported goods to maximize an intertemporal utility function. Household income is consumed or saved in the form of equity in domestic firms or foreign bonds. Home firm equities and foreign bonds are assumed to be perfect substitutes, and the private agent has access to the world capital markets freely at a given world interest rate. In this environment, the representative household owns labor and all financial wealth, and allocates income to consumption and savings to maximize an intertemporal utility over an infinite horizon, as culminated into Equation (2.??). The form of the instantaneous felicity adopted here is a simple logarithmic function in which the consumption aggregate, TC_t is the argument, i.e.,

$$u_t = \ln(TC_t). \qquad (3.15)$$

It is known that the logarithmic form is a subset of the constant intertemporal elasticity function, with σ=1. Thus, the *Euler* condition for the optimal consumption path of the household takes the following form:

$$\frac{Ptc_t}{Ptc_{t-1}} \cdot \frac{TC_t}{TC_{t-1}} = \frac{(1+r_t)(1+n)}{(1+\rho)}. \qquad (3.16)$$

Here, TC_t is instantaneous aggregate consumption generated from final goods. The *Ptc* is the consumer price index such that

$$Ptc_t TC_t = \sum_i PC_{it} C_{it} \qquad (3.17)$$

with PC_{it} denoting the (composite) price of commodity-i; and C_{it} denoting the sectoral consumption demand.

As shown in Chapter II, the steady state equilibrium assumption necessarily imposes the condition, ρ=r+n, since Ptc and TC are constant in the steady state. In what follows, we will assume for simplicity that n=0; or alternatively, one can interpret all variables of the system in *per capita* terms.

At the *second level* of the consumer's problem, the (intertemporally optimal) aggregate consumption is dispersed among its sectoral components. A simple functional form to achieve this allocation is that of Cobb-Douglas with,

$$TC_t = \prod_{i=1}^{m} C_{it}^{b_i} \qquad (3.18)$$

where m is the number of sectors (m=2); and b_i are the consumption shares which are already discussed in the previous section, i.e., they are obtained from the data spelled out in the SAM.

On the production side, one important contrast to the analytical model is that a new piece of capital is produced by the two final goods via a constant returns to scale technology. We also assume that there are no additional capital installation costs beyond the cost of the final goods used in the production of the capital good. Hence, at equilibrium with a positive level of investment, the unit cost of capital investment, denoted *PI*, is uniquely determined by the prices of the final goods. Following the practice of normalizing all prices to unity, *PI* is also normalized to one. Once *PI* is normalized to one, the quantity of investment, *INV*, can be obtained from the Social Accounting Matrix (SAM) data in the cells recording the flows between commodity activities and capital accumulation activities

As discussed more thoroughly in Chapter II, under the assumptions of perfectly substitutability among assets, and the presence of perfectly efficient capital markets, capital returns should be equal to the rate of interest without depreciation or equal to interest rate plus depreciation rate, *dpr* , if dpr is positive, i.e.,

$$wk = r + dpr. \qquad (3.19)$$

where wk is capital return rate. In Section II.4 of Chapter II, we assumed that capital is the forgone output from Sector 2 only, and the price for this sector's output was normalized to one. In contrast to that specification, when we assume that a new increment of capital is produced by the both of the sectors' forgone outputs, the unit cost of capital production, PI, is determined by both sectors' prices. Thus, in the steady state equilibrium, the following relationship between interest rate and capital return has to be held:

$$\left(r_{SS} + dpr\right) = \frac{Wk_{SS}}{PI_{SS}}. \qquad (3.20)$$

where time subscript *SS* stands for the steady state values of the relevant variables. At this stage, a choice for a value for the interest rate, r_{SS}, can be from outside information. An estimate of the household's rate of time preference, ρ is also needed. The value of PI is known and normalized to one. Since both wk_{SS} and *dpr* cannot be obtained from this single equation, additional information from the SAM is required. The SAM provides information on the total returns to capital, i.e., $Wk_{SS}K_{SS}$. The task for us is to separate wk_{SS} from K_{SS}. For this purpose, both sides of the Equation (3.20) are multiplied by K_{ss} and obtain,

$$(r_{ss} + dpr)K_{ss} = \frac{wk_{ss}K_{ss}}{PI_{ss}}. \qquad (3.21)$$

Then, we substitute for $K_{ss} = \dfrac{INV_{ss}}{dpr}$, and rearrange to obtain the rate of depreciation associated with the steady state of the economy:

$$dpr = \frac{r_{ss}PI_{ss}INV_{ss}}{\left(Wk_{ss}K_{ss} - PI_{ss}INV_{ss}\right)} \qquad (3.22)$$

The products, $Wk_{SS}\times K_{SS}$ and $PI_{SS}\times INV_{SS}$ in equation (3.22) above are obtained from the respective SAM directly (as aggregate returns of the capital factor). To distinguish the steady state capital rental price from the quantity of initial capital stock, we utilize Equation (3.20) once again to obtain:

$$wk_{SS}= (dpr+r_{SS})PI_{SS}. \qquad (3.23)$$

From this calculation, we obtain an estimate of the initial capital stock using the SAM data on $Wk_{SS}K_{SS}$.

The initial level of the trade deficit is also given by the SAM data. If data suggest an initial trade deficit, then to satisfy the steady state condition, (2.XX), foreign debt must be a negative quantity (that is, we are now dealing with *foreign assets*). To see this point more clearly, note that the SAM data reflects a deficit position in the Turkish commodity trade by TL 16,972.807 billions in 1990 prices. We infer that the interest revenue from the previously accumulated foreign assets owned by the domestic residents finances the ongoing trade deficit. That is, if we assume an interest rate of 11%, we obtain from SAM:

$$D_{SS} = -\frac{FSAV_{SS}}{r_{SS}} = -\frac{16972.807}{0.11} = -154298.245$$

At first glance this reflects a counterfactual state of affairs, since most developing countries including Turkey have accumulated a stock of actual debt, and are simultaneously running trade deficits. Note, however, that the D_{SS} is to be interpreted as a technical variable to impose the equilibrium constraints of the theory, rather than estimates of actual debt/asset position of the domestic economy. Starting from the combination D_{SS} and $FSAV_{SS}$, when the equilibrium is perturbed by alternative policy environments, any depletion of initial period assets will be interpreted as accumulation of foreign debt *relative* to the base-run steady state path. Thus, this interpretation reinforces, once again, the basic principle underlying the motivations behind the construction and use of general equilibrium modeling techniques: that these are an "economics laboratory" devices, and any of the policy experiments performed are basically of *comparative* nature and are typically meaningful only in relation to each other, rather than revealing forecasts of the future.

******* a conclusion needed to link with the upcoming material*****

FILE: ch3-4-2.doc (June 26-pm); this is a slight revision of file ch3-mod1.doc

III-4. A simple dynamic CGE model with imperfectly substitute commodities in trade

Now we give a full description of the simple dynamic CGE model (to which we give the acronym, DCGE) of the Turkish economy in its algebraic formation, and then we will use it to discuss the transitional dynamics with the introduction of the imperfectly substitute commodities in trade.

It is often convenient to specify a DCGE model in terms of three blocks of equations, each of which describing a particular aspect of the equilibrium conditions. These are: the intratemporal equilibrium block, the dynamic equilibrium block, and the steady state equilibrium block.

III-4-1. The intra-temporal equilibrium block

The structure of the intratemporal equilibrium block is comparable to a static CGE model. In this block, a set of price equations is specified which links world market prices with domestic prices, a set of demand systems derived from the first order conditions of the consumer's and producer's problem, and a set of within-period market clearing conditions. We discuss each of these in turn:

The price system and foreign trade

Although world prices are treated as parameters to the economy, the two-way trade flows appear in the data of each sector due, in part, to the relatively high degree of sectoral aggregation or relatively heterogeneous goods. This problem is treated as though foreign and domestic goods within the same sector are imperfect substitutes. The extent of imperfect substitutability among the goods is expressed by--what we refer to as the *Armingtonian composite commodity* system. In this system, domestically produced goods and imports in the same sector category are differentiated by geographical origin of production. As discussed in equation (3.1) the sectoral composite good, *CC*, is formulated as a CES aggregation of the domestic commodity, *DC*, and the imported foreign good, *M*.

WE SHOULD SUBSCRIPT CC, AND DC etc AND NOTE I=2 or make clear we are given example of one sector

Treating *DC* and *M* as "inputs" to be "employed" to generate the composite good, *CC*, the demanders are then hypothesized as minimizing a cost function:

$$PD \cdot DC + PW(1+tm) \cdot M \qquad (3.24)^{*}$$ [4]

subject to the CES composite commodity "technology", Eq. (3.24), The first order conditions of this problem become:

$$\frac{M}{DC} = \left[\frac{PD}{PW(1+tm)}\right]^{\sigma m} \left[\frac{\beta}{1-\beta}\right]^{\sigma m} \qquad (3.25)$$

which sets the sectoral import demand function as an optimal ratio of *M* to *DC* given the respective prices and the structural parameters. In Equations. (3.25) and (3.26), the world price, *PW*, which is an exogenous variable, is distorted by the sectoral tariff rate, *tm*, a policy instrument parameter; while the price of the domestic good, *PD*, is solved endogenously within the general equilibrium system to satisfy the market clearing conditions.

The composite good price, *PC*, is expressed as the dual of the CES aggregation:

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[4] In what follows, we will adopt the convention that the equation number with a "*" indicates that this equation is not used in the algebraic equation system written in the computer software --the GAMS code for solving the model, while the equation's number without * implies that this equation enters as an explicit equation for solving the model.

$$PC = \frac{1}{AC}\left[\beta^{\sigma m}(PW(1+tm))^{1-\sigma m} + (1-\beta)^{\sigma m} PD^{1-\sigma m}\right]^{\frac{1}{1-\sigma m}} \qquad (3.26)$$

and hence is an endogenous variable.

On the export side, the hypothesis of product differentiation is carried over to the producers. As discussed in Eq. (3.6), the producer in each sector is envisaged to face a transformation frontier on his market sales: domestic versus export. Just like the demanders, the producer is considered to solve an optimization problem of maximizing revenues from both markets, i.e.,

$$PD \cdot DC + PW \cdot E \qquad (3.27)^*$$

subject to the transformation technology, Equation (3.28). The solution to this problem yields the export supply function as the optimal ratio of E to DC given respective prices, the structural share parameter, *h*, and the transformation elasticity, *se*:

$$\frac{E}{DC} = \left[\frac{PW}{PD}\right]^{\sigma e}\left[\frac{1-\eta}{\eta}\right]^{\sigma e} \qquad (3.28)$$

The unit revenue function derived from gross sales can then be specified as the dual of the revenue maximization problem:

$$PX = \frac{1}{AT}\left[\eta^{-\sigma e}(PW)^{1+\sigma e} + (1-\eta)^{-\sigma e} PD^{1+\sigma e}\right]^{\frac{1}{1+\sigma e_i}} \qquad (3.29)$$

To complete the price system, we introduce the unit price of value added, *PV.* The value added price discloses the net revenue to the producer --net of intermediate input costs and of producer taxes:

$$PV = PX(1-tx) - \sum_j PC_j a_{ji} \qquad (3.30)$$

where tx denotes the producer tax rate, another policy instrument parameter. The intermediate costs are spelled by the expression under the summation term where a_{ji} denotes the input/output coefficient for good j, used by sector i.

Factor demand and factor market clearing

We posit a neoclassical production function to reflect the relationship between inputs and the gross output. We identify two primary inputs, capital and labor. Conceptually, we separate the intermediate input use from that of production of the value added component. For intermediate input demands, *INT*, we make use of the fixed input-output coefficients, as would be derived from a Leontieff technology:

$$INT_i = \sum_j a_{ji} XS_i \qquad (3.31)$$

As for *value added*, a simple functional form which accommodates many relevant attributes of the factor markets is that of *Cobb-Douglas*. It is known that the Cobb-Douglas form is a special case of the CES technology with the elasticity of substitution between capital and labor to be 1.0. Thus, gross output is linked to the primary factors, K and L, via,

$$XS = AX \cdot L^{\alpha} K^{(1-\alpha)} . \qquad (3.32)$$

Given the wage rate, Wk, and capital rental rate, Wl, the first order conditions of profit maximization state that the value of marginal product of labor and capital should be equated to Wl and Wk, respectively, i.e.

$$Wl = \frac{\alpha PV \cdot XS}{L}$$

$$Wk = \frac{(1-\alpha) PV \cdot XS}{K} \qquad (3.33)$$

Given the demand for labor, L, and capital, K, as such, the labor and capital market equilibrium condition sets the equilibrium capital rental rate and wage rate from:

$$\sum_i L_i = L^S$$

$$\sum_i K_i = K^S \qquad (3.34)$$

where L^S and K^S are the aggregate supply of labor and capital, respectively. Note that in DCGE supply of capital, K^S, is a time-dependent endogenous variable, while the supply of labor, L^S, is an exogenous variable, and is regarded time-independent. We postpone the discussion of the supply of capital to discussion of the dynamic block of equations, as capital being the cumulative factor, is subject to additional constraints involving time..

Income Generation and Commodity Demand

The current period flow of private income is generated from three sources: factor income from ownership of labor and capital; interest income/expenditures on lending/borrowing from abroad; and transfers of distortionary tax revenues. Thus,

$$YH = \left[Wl \cdot L^S + Wk \cdot K^S \right] - r\, D + TRSFER , \qquad (3.35)$$

where YH denotes total household income, D is the outstanding foreign debt (if negative it becomes foreign assets), and $TRSFER$ denotes the total tax revenues collected through the tax administration. In this simple model, we do not identify any behavioral rules for the public sector and abstain from macro issues on public deficits and/or government debt. Thus, we accommodate the tax system only to the extent that they affect the domestic relative price system, and all the government tax revenues are rebated lump sum back to the consumer:

$$TRSFER = \sum_i tm_i PWM_i M_i + \sum_i tx_i PX_i XS_i \qquad (3.36)$$

This specification enables us to focus on issues of resource allocation efficiency, while postponing discussion of macro phenomena such as the public savings-investment gap, management of the public borrowing requirements, etc. at a later stage. These issues are to be taken separately in Chapter IV.

The foreign debt variable, D, is a time-dependent, variable and will be further addressed in the dynamic blocks. Aggregate consumption demand, TC, is derived from the Euler equation described in (2.??) and will likewise be narrated in the dynamic blocks.

The difference between total income and total consumption expenditure in each time period is the household savings, SAV, for that period, i.e.,

$$SAV = YH - Ptc \cdot TC, \qquad (3.37)$$

where Ptc is a price index over the composite good prices, PC. Given the aggregate consumption expenditure which is yet determined in the dynamic block, the household demand for an individual commodity is:

$$CD_i = cles_i \frac{Ptc \cdot TC}{PC_i} \qquad (3.38)$$

where $cles_i$ denotes sectoral share parameters with $\sum cles_i = 1.0$.

Investment expenditure is determined in the dynamic blocks, while demand for each final good employed in the formation of a new piece of capital can be defined as follows:

$$ID_i = iles_i \frac{PI \cdot INV}{PC_i} \qquad (3.39)$$

where $iles_i$ is the sectoral share parameter wýth $\sum iles_i = 1.0$.

For each sector, the product market equilibrium satisfies the sum of consumption demand, *CD*, investment demand, *ID*, and the intermediate demand, *INT*, has to be equal to the sectoral absorption; i.e., the supply of the composite good, *CC*:

$$CD_i + ID_i + INT_i = CC_i \qquad (3.40)$$

The model relies on the price of the domestic good, *PD*, as the endogenous variable to satisfy this set of market constraints. Observe that, in the terminology of the SAM accounting of Section III-2 this equation depicts the "Commodities" row of the SAM referred in Table III-2.

Finally, we close the within-period block of equations of our simple model by imposing the Balance of Payments constraint:

$$\sum_i PW_i \cdot M_i - \sum_i PW_i \cdot E_i = FSAV \qquad (3.41)$$

where the residual trade balance, *FSAV*, is met by foreign borrowing and will also enter into some of the dynamic equations to be defined below in the dynamic blocks. Equation (3.41) reflects the row and column balance of the "Trade" account of the SAM.

Our dynamic setting contrasts with a static model in its explicit accommodation of the time dimension for each variable and equation. As the time paths for all variables are solved simultaneously, the number of equations/variables increase in proportion to the number of time periods. The linkages between time periods are built into the intertemporal equilibrium block.

III-4-2. The intertemporal equilibrium block

Intertemporal behavior equations for households

The first order conditions for the household intertemporal problem are derived as the Euler equations defined in Eq. (2.8). By choosing a logarithmic functional form for the intra-temporal felicity function, one can define a set of difference equations which links the two consecutive time periods' aggregate consumption, *TC*, with their respective prices, *Ptc*, as follows:

$$\frac{Ptc_t}{Ptc_{t-1}} \cdot \frac{TC_t}{TC_{t-1}} = \frac{(1+r_t)}{(1+\rho)} \qquad (3.42)$$

(note that we assume away population growth in this simple model).

This difference equation system, together with the current budget constraint in Equation (3.35) and the transversality condition serve to define the level of the aggregate consumption, *TC*, and savings, *SAV*, for the each time period.

Intertemporal behavior equations for investment

Even though we assume that the households own the stock of capital, it is convenient to separate the investment behavior from household's consumption/saving decisions by constructing an independent investor that is presumed to maximize the intertemporal profits of capital investment. This setup is adapted from Wilcoxen (1988) and Ho (1989). Formally, we assume that an artifact bank chooses quantity of investment to maximize

$$\sum_{t=1}^{\infty} R_t (Wk_t K_t - VI_t) \tag{3.43}$$

subject to

$$K_{t+1} = (1 - dpr)K_t + INV_t, \tag{3.44}$$

where R_t is a discount factor defined as

$$R_t = \prod_{s=0}^{t} \frac{1}{(1 + r_s)},$$

VI_t is the value of investment at t; INV_t is the new physical capital good, and *dpr* is the (constant) rate of capital depreciation. Incorporating the assumptions that the technology to produce capital exhibits constant returns to scale, and that there are no additional capital installation costs beyond the direct costs of the final goods employed in capital production, then, at equilibrium with a positive level of investment, the value of each unit of capital is uniquely determined by the prices of the final goods. Thus,

$$VI_t = PI_t INV_t \tag{3.45}$$

where PI_t is the cost for each unit of INV_t, and is uniquely determined by the price for the composite goods, i.e.,

$$PI = \prod_i PC_i^{iles_i} \tag{3.46}$$

The *no-arbitrage* condition derived from this problem becomes,

$$r_t PI_{t-1} = wk_t - dpr \cdot PI_t + \Delta PI_t \tag{3.47}$$

This difference equation indicates that the total returns to a unit of capital derived in the right-hand-side of the equation equals the return to a perfectly substitutable asset of size PI_{t-1}.

It should be noticed that without the assumption of imperfect substitution between the domestically produced and consumed good, and the foreign good (the Armington specification), both *PI* and *Wk* are determined by the world output prices. In this case, both *PI* and *Wk* are time invariant given exogenous world prices. It is for this reason that in the small open economy model of Chapter II, we cannot derive this independent intertemporal investment decision function. Only when the Armington specification is introduced into the model, can the investment decision, as described by this equation, be treated as an independent activity. This equation, together with the transversality condition for foreign debt, as defined in the next block, serve as a set of difference equations to derive the demand for the total investment, *INV*, in each period. In the next subsection, we will further isolate the importance of Armington specification on the functions describing investment decisions.

The accumulation of capital

Since the stock of capital evolves over time along the transition path, the technological constraint for the investment problem defined in Eq. (3.44) is introduced into the dynamic equilibrium system explicitly. This equation serves to link the current period capital stock and investment with the next period capital stock.

The accumulation of foreign assets/debt

In each time period, the difference between the value of capital investment, PI_tINV_t, and the domestic savings, SAV_t, is covered by borrowing from abroad (foreign savings, $FSAV_t$) which is represented by the trade deficit. Accumulated trade deficits become the stock of foreign debt. Hence, the dynamic equilibrium system has to include an equation to represent such a relationship:

$$D_t - D_{t-1} = r_t D_{t-1} + FSAV_t \qquad (3.48)$$

This equation implies that an increase in the foreign debt has two components: a current period trade deficit, denoted as $FSAV_t$, and the interest costs on outstanding foreign debt, r_tD_{t-1}. This difference equation serves to link the current period trade deficit and outstanding debt with the next period foreign debt.

III-4-3. The steady state equilibrium block

In an infinite horizon dynamic model, the transversality conditions are satisfied in the limit as time goes to infinite. Of course, a numerical discrete time model must eventually be terminated in finite time. The terminal period, *T*, is determined when the steady state equilibrium conditions are approximately satisfied. Thus, the equations describing the steady state equilibrium conditions are introduced in period *T*, when the model is terminated. Similar to the transversality constraint in an infinite horizon problem, the steady state equilibrium conditions here serve as a set of intertemporal constraints to avoid unlimited foreign borrowing by households or investors.

The first steady state constraint equation is derived from the *non-arbitrage* condition for investment:

$$r_{SS} + dpr = Wk_{SS} / PI_{SS} \tag{3.49}$$

When a steady state is approached, this equation requires that returns to capital, *Wk*, and the unit cost of investment, *PI*, become constant and the ratio of these two variables must equal to the interest rate (a constant) plus the capital depreciation rate.

The second steady state constraint is derived from the capital accumulation equation:

$$INV_{SS} = dprK_{SS} \tag{3.50}$$

which implies that the stock of capital becomes constant and investment just covers the depreciated capital.

The last steady state constraint is a budget constraint for the economy, that is, its foreign debt has to be constant:

$$FSAV_{SS} + r_{SS}\, D_{SS} = 0 \tag{3.51}$$

In the steady state, this constraint requires that foreign borrowing is negative if the outstanding foreign debt for the economy is positive (a positive *D*). In this case, the economy has to run a trade surplus in perpetuity to pay the interest costs on the outstanding debt, i.e., $FSAV_{SS}$ has to be negative.

It should be noticed that the steady state equilibrium conditions depict more than a single time period; rather, they refer to an equilibrium path in which all endogenous variables approach to constant values in per capita terms. In contrast to the out-of-steady state paths, all endogenous variables thus become time-independent in the steady state.

This time invariance property allows us to define a steady state in the model terminal period.

Equations defined in each of the three blocks can be easily seen to correspond on a one-to-one base with those in the theoretical model of Chapter II, except for the equations that account for the three major revisions made to the theoretical model. In the intra-temporal equilibrium block, the Armington/CET specifications are introduced to capture the two way trade phenomenon observed in the data; in the intre-temporal aquarium block, the non-arbitrage condition for capital investment is introduced to derive the intertemporal investment behavior; and finally, under the steady state equilibrium block the steady state constraints are introduced explicitly to assure that the model satisfies the transversality conditions.

Solving the equations under the three blocks simultaneously permits the use of commercial software such as GAMS. Solving the equations simultaneously is required in this case because–(as shown in Chapter II), the current decisions of the households or investors not only depend on the current income/profits and prices but also on the future income/profits and prices. Thus, each time period's equilibrium cannot be independently defined without equilibria being resolved in other time periods. Moreover, the transitional equilibrium cannot be solved without defining the steady state equilibrium.

The model solution for each variable described above should exactly equal its initial value in the SAM, if all of the parameters in the model have been correctly calibrated, as discussed in the previous section, presuming policy instruments remain at the levels observed in the SAM. Furthermore, as we assume the data in the SAM represent an initial steady state equilibrium, all the variables should be constant over time. We describe such corresponding relationships between the model variables and the data of the SAM in Tables III-4 and III-5.

Policy experiments (simulations) often entail parametric changes in the level of the government's policy instruments from their levels appearing in the SAM, and then solving the model to determine the effect of these changes on the path of the model's endogenous variables.

III-5. Instanteous Convergence versus Out-of- Steady State Dynamics

The property of instantaneous adjustment to a steady state noted for the small open economy studied in Chapter II is undesirable for policy purposes since it appears counter factual. Moreover, in this case, a modeled economy can only have a single equilibrium which is a steady state equilibrium given the world price and the interest rate. The economy cannot adjust from its present equilibrium to another for any parametric change in a policy instrument which affects relative price in the domestic economy. The reason is that the ratio of the capital rental rate over the unit cost of capital production, *Wk/PI* is only a function of the relative output price, which is equal to the world price but is distorted by, e.g., the country's tariff rates. In the steady state, this ratio equals the constant world interest rate (plus a constant depreciation rate), as defined in Eq. (3.49). Given a constant interest rate, if the relative output price changes because the modeler makes a parametric change in the tariff rate, the equality requirement defined in Eq. (3.49) is violated for all time, and hence, the economy can never reach to a new equilibrium.

Many efforts have been made to overcome this shortcoming of the basic small open economy. The widely adopted methods are to incorporate adjustment costs in capital investment, as in (cite reference here) or to introduce an imperfect capital market for the economy as in (cite). In the above algebraic model structure, we introduce another possible choice to restore the transitional dynamics for a small open economy through the specifications of Armington/CET system.

When the home produced good is treated (via the Armington or CET functions) as an imperfect substitute for the foreign produced good in the same sector category, the domestic price system tends to depart from the world price system. That is, prices become endogenously determined. Now, the households and the investor face a set of prices for the composite good, *PC* (see Eq. (3.26)), while the producer prices are *PX* (see Eq. (3.29)) for the composite output. Both of these prices are a function of *PD* --the price for the good, *DC*, produced and consumed domestically. *The market clearing condition that the demand for DC has to be equal to the supply of DC, i.e., determines PD*,

$$AC^{1+\sigma m}\left[(1-\beta)\frac{PC}{PD}\right]^{\sigma m} CC = AT^{-(1+\sigma e)}\left[(1-\eta)\frac{PX}{PD}\right]^{-\sigma e} XS\,, \qquad (3.52)$$

where the left-hand-side of the equation is demand for *DC,* while the right-hand-side is the supply of *DC*. This function can be treated as an implicit function for *PD* such that

$$PD = PD(CC, XS; PW)\,. \qquad (3.53)$$

Total demand for the composite good, *CC*, is the summation of household demand, *CD*, which depends on the household income over the path to the steady state, intermediate input demand, *INT*, (which has a fixed ratio to the output), and investment demand, *ID*, (which depends on the investment decision). The supply of output, *XS*, depends on the supply of labor and capital. Hence, *PD* is ultimately a function of the stock of capital, *K*, and investment, *INV*, i.e.,

$$PD = PD(K, I; PW, L)\,. \qquad (3.54)$$

Once price *PD* becomes a function of capital and investment, so do *PC* and *PX* which are functions of *PD*. In Eq. (3.46), it is obvious that the unit cost for capital investment, *PI*, is a function of *PC*, and hence a function of capital and investment. Also, the labor and capital rental rates, *wl* and *wk* can be derived as functions of *PX* by Eqs. (**??**) in Chapter II, and hence *wl* and *wk* are functions of capital and investment. Now, return to the non-arbitrage condition for the investment. Note that it can be rewritten in following form:

$$r_t PI(K_{t-1}, I_{t-1}) = wk(K_t, I_t) - dpr \cdot PI(K_t, I_t) + \Delta PI(K_t, I_t) \qquad (3.55)$$

Then, the non-arbitrage condition for investment becomes an implicit difference function over investment, *INV* and stock of capital, *K*, and hence can be treated as an implicit investment demand function.

Under this setup, when the world price, *PW*, or trade policy changes, *wk* and *PI* cannot jump to their steady state level since the current level of capital stock affects their determination. The capital and investment adjust slowly until ΔPI becomes zero, and the ratio of *wk/PI* becomes constant and equals to $r + dpr$ again, as Eq. (*) requires. Thus, this specification of the model can be viewed as treating all goods as quasi-home goods.

In order to see this picture more clearly, we conduct a numerical example as follows. First we reorganize the data presented in the Turkish SAM such that the home and foreign produced goods are perfectly substitutes. We then shock this economy using the algebraic model described above, but ignoring the Armington/CET specification in the intra-temporal equilibrium block. The experiments we conduct are to increase and to decrease the capital stock. Then, in solving the model, the non-transitional property is easily observed, i.e., the level of capital stock jumps to the steady state immediately by borrowing or lending from abroad. We show this result in the following table (**??**). In the first period, capital is reduced by 5 percent from its steady state level. Then we observe that investment increases by 16.2 percent such that the level of capital stock jumps to its steady state level in the next period. To finance this increase in investment, the trade deficit increases by 101 percent in the first period due to foreign borrowing. Hence in the new steady state, the only difference from the former is that the economy has to run a trade surplus to pay its debt caused by the first period's borrowing. All other variables remain unchanged.

Now return to the Turkish SAM and shock the economy in the similar way using this model, but with the Armington/CET specification. It can be seen that transition paths to a new steady state obtains.

Since, the Armington/CET specification is the only reason for the model to display transitional dynamics, logically, we should believe that the convergent speed of the model depends upon the elasticities of substitution between goods produced at home and abroad, and the size of the shock. For a given shock, if home goods easily substitute for foreign goods, then the resultant changes in the key dynamic variables, such as investment and debt, along transitional paths are larger, and the new steady state is "approached" in a relatively shorter time. If, instead, the home good is a poor substitute for foreign goods, then, the transitional dynamics are more protracted.

Next, we show the effects of the elasticity of substitution on the pace of convergence to a steady state using the same model. For alternative Armington/CET elasticities, the model is solved with all tariffs set to zero. The simulation results showing the transitional paths of four dynamic variables, investment, stock of capital, trade deficit and foreign assets (negative of the foreign debt), are depicted in Figures III-? through III-?, while the terminal periods when the new steady state is approached approximately are in Table III-?. We rely on the following indicators to investigate the convergent speed towards a new steady state: one is the time horizon when 99.99 percent of the transitional stage of the main variables is realized; and the second one is the time period when all endogenous variables cease to change by less than 0.000001. For various elasticities, the paths of transition and the associated convergence periods are diverse. We observe that, when tariffs are eliminated, the lower the substitution possibilities between the foreign and the own goods, the "flatter" is the path of the endogenous variables to their steady state values, i.e., the convergent speed to the steady state increases (see Table III-?).

Table III-1. Structure of A Simple SAM for an Open Economy					
	Production Activities	**Market Activities**	**Agent**	**Accumulation Activities**	**Trade Activities**
Production Activities		Domestic Sales, DC			Exports, E
Market Activities			Consumption, C	Investment, I	
Agent	National Income, Y				
Accumulation Activities			Savings, S		Trade Deficit, TDEF
Trade Activities		Imports, M			

Table III-2. Structure of A More Detailed Social Accounting Matrix									
	Production Activities	**Commodities**	**Labor**	**Capital**	**Households**	**Government Policy**	**Accumulation**	**Trade**	**Total Receipts**
Production Activities		Domestic Supply of DC						Exports	Output Supply
Commodities	Intermediate Inputs				Private Consumption		Investment Expenditures		Domestic Absorption
Labor	Wages								Labor Income
Capital	Returns to Capital								Capital Income
Households			Wages	Returns to Capital		Transfers of Tax Revenues			Private Income
Government Policy	Production Taxes	Tariffs							Tax Revenues
Accumulation					Savings			Foreign Savings	Private Savings
Trade		Imports							Foreign Exchange Spent
Total Expenditures	Production Costs	Aggregate Absorption	Labor Income	Capital Income	National Income	Tax Revenues Disposed	Aggregate Investment	Foreign Exchange Earnings	

Table III-3. Aggregated SAM for Turkey, 1990 (Billions TL)

		Activities		Commodities		Factors		Agents		Capital Acc		
		1.Agr	2.Ind	3.Rural	4.Urban	5.Labor	6.Capital	7.Private	8. Gov.	9.Investment	10.ROW	SUM
Activities	1. Agriculture			93927.092	0.000						2513.039	96440.131
	2. Industry			0.000	522787.012						49548.516	572335.528
Commodities	3. Rural	14926.387	25134.268					52600.620	346.972	3998.087		97006.334
	4. Urban	16088.999	235097.972					209605.316	42736.494	98610.192		602138.973
Factors	5. Labor	37166.906	132994.932									170161.838
	6. Capital	28885.165	157966.714									186851.879
Agents	7.Private					170161.838	173401.601					343563.439
	8.Government	-627.326	21141.642	469.059	12927.782		13450.278	14568.474				61929.909
Capital	9.Savings							66789.029	18846.443		16972.807	102608.279
ROW	10.ROW			2610.183	66424.179							69034.362
TOTALS		96440.131	572335.528	97006.334	602138.973	170161.838	186851.879	343563.439	61929.909	102608.279	69034.362	

Table III-4. Aggregated SAM of The Simple Dynamic General Equilibrium Model

				Factors		Institutions		Capital Acc		
		1.Activities	2.Commodities	3.Labor	4.Capital	5.Private	6. Government	7.Investment	8.ROW	SUM
	1.Activities		$\sum PD_i DC_i$						$\sum PWE_i E_i$	*Value of Output*
	2.Commodities	$\sum_{ij} a_{ji} PX_i XS_i$				$\sum PC_i CC_i$		$\sum PC_i ID_i$		*Absorption*
Factors	3. Labor	*Wl x L*								*Labor Income*
	4. Capital	*Wk x K*								*Capital Income*
Institutions	5.Private			*Wl x L*	*Wk x K*		*YG*		*FSAV*	*Private Income*
	6.Government	$\sum tx_i PX_i XS_i$	$\sum tm_i PW_i M_i$							*Government Income*
Capital Account	7.Savings					*SAV*				*Aggregate Savings*
ROW	8.ROW		$\sum PW_i M_i$							*Foreign Exchange Spent*
TOTALS		$\sum PX_i XS_i$	$\sum PC_i CC_i$	*Wl x L*	*Wk x K*	*YH*	*YG*	*Aggregate Investment*	*Foreign Exchange Earnings*	

Table III-5. Equilibrium in the Social Accounting Matrix Balances			
ACCOUNT	**RECEIPTS (ROW)**	**=**	**EXPENDITURES (COLUMN)**
Activities	PD*DC + PWE*E	PX*XS	INT + WI*L + Wk*K + tx*PX*XS
Commodities	PC[INT + CD + ID]	Pc*CC	PD*DC + tm*PWM*M + PWM*M
Households	WI*L + Wk*K + FSAV$_{SS}$ + YG	YH	PC*CD + SAV$_{SS}$
Government	tx*PX*XS + tm*PWM*M	=	YG
Capital Account	SAV$_{SS}$	=	PC*ID
Rest of the World	PWM*M	=	PWE*E + FSAV$_{SS}$