1 Product Differentiation in CGE Modeling

1.1 The Armingtonian Commodity System

Motivation: Empirical data suggests that, even at higher levels of commodity disaggregation, "two-way trade" is observed. That is, at the sectoral level, we observe both imports and exports.

Models built on "the law of one price" generates extreme specialization across sectors, with implausibly large swings in domestic relative prices and sectoral reallocation of resources in response to trade policy instruments and/or changes in the world prices.

Consumers regard import and the domestically produced good as an *imperfectly substitute* of each other. Given the relative price of "import" to that of the "domestically produced good" they choose the optimal import to domestic good consumption ratio so as to minimize their expenditures.

Producers regard export and the domestic sales as imperfectly substituting each other along a *constant elasticity frontier* (the production possibility boundary of the economy). Given the relative price of export to domestic good, they choose the optimal export to domestic sales ratio so as to Max their revenues.

The *real exchange rate* in this system is differentiated by the end user. For *consumers* $ER^R = \frac{PM}{PD}$; whereas for the *producers* $ER^R = \frac{PE}{PD}$.

1.2 Specification of Foreign trade under Product Differentiation

Consumers

Composite good aggregation function (CES)

$$CC = \overline{AC}\left[\delta M^{-\rho} + (1-\delta)D^{-\rho}\right]^{-1/\rho} \tag{1}$$

with elasticity of substitution between M and D, $\sigma = \frac{1}{1+\rho}$

Domestic price of imports

$$PM = (1+tm)ER \cdot PWM \tag{2}$$

Consumers minimize their expenditures on imports and domestic purchases $PM \cdot M + PD \cdot D$ subject to(1) The solution is the import demand function:

$$\frac{M}{D} = \left(\frac{PD}{PM}\right)^{\sigma}\left(\frac{\delta}{1-\delta}\right)^{\sigma} \tag{3}$$

Producers

They have the production technology to produce $XS = f(K, L)$. given the production of XS, producers have to decide to market their sales to domestic market or to exports. Given their production possibility frontier (CET)

$$XS = \overline{AT}\left[\beta E^{\gamma} + (1-\beta)D^{\gamma}\right]^{1/\gamma} \tag{4}$$

and given the domestic price of export

$$PE = (1+se)ER \cdot PWE \tag{5}$$

they maximize revenues. To do so they maximize $PD \cdot D + PE \cdot E$ subject to (4) to obtain the export function

$$\frac{E}{D} = \left(\frac{PE}{PM}\right)^{\Omega} \left(\frac{1-\beta}{\beta}\right)^{\Omega} \tag{6}$$

with $\Omega = \frac{1}{\gamma - 1}$.
Balance of payments satisfy

$$PWM \cdot M - PWE \cdot E \quad = \quad FSAV \tag{7}$$

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$$ER = 1 \tag{8}$$

Endogenous Variables		Exogenous variables	
E	exports	XS	output supply
M	Imports	PWE	world price of exports
D	domestic good sold domestically	PWM	world price of imports
CC	Composite good (absorption)	FSAV	Trade balance (foregin savings)
PE	domestic price of exports	Ω	transformation elasticity
PM	domestic price of imports	σ	substitution elasticity
Pd	price of the domestic good	ER	exchange rate

1.3 GAMS code

```
*##TRADE QUANTITIES
    M0(I) = MATBAL("IMPORTS",I) + MATBAL("TARIFFS",I) ;
    E0(I) = MATBAL("EXPORTS",I) + MATBAL("EXTAX",I) ;
    DC0(I) = (PX0(I)/PD0(I))*XS0(I) - (PE0(I)/PD0(I))*E0(I);
    CC0(I) = (PM0(I)*M0(I) + PD0(I)*DC0(I))/PC0(I) ;
    *##WORLD PRICES AND COMMERCIAL INSTRUMENTS
    TM(I) = MATBAL("TARIFFS",IM)/MATBAL("IMPORTS",IM) ;
    TE(I) = MATBAL("EXTAX",IE)/MATBAL("EXPORTS",IE) ;

    PWM(I) = PM0(I)/(1+TM(I)) ;
    PWE(I) = PE0(I)/(1-TE(I)) ;

    DISPLAY TM, TE, PWM, PWE, E0, M0 ;
```

```
*#*#*CALIBRATION OF ALL SHIFT AND SHARE PARAMETERS
*
*##FOREIGN TRADE FUNCTIONS
RHOC(I) = (1/MATBAL("SIGC",I) ) - 1 ;
RHOT(I) = (1/MATBAL("SIGT",I) ) + 1 ;
BC(I) = PM0(I)/PD0(I)*(M0(I)/DC0(I))**(1+RHOC(I))
BC(I) = BC(I)/(1 + BC(I) ) ;
```

that is, from (3) solve for

$$\begin{aligned}\left(\frac{\delta}{1-\delta}\right)^{\sigma} &= \left(\frac{PM}{PD}\right)^{\sigma}\left(\frac{M}{D}\right)\\ DELTA &= \left(\frac{\delta}{1-\delta}\right) = \left(\frac{PM}{PD}\right)\left(\frac{M}{D}\right)^{1/\sigma}\end{aligned}$$

using $\sigma = \frac{1}{1+\rho}, 1/\sigma = 1+\rho$. Thus, $DELTA$ becomes

$$DELTA = \left(\frac{\delta}{1-\delta}\right) = \left(\frac{PM}{PD}\right)\left(\frac{M}{D}\right)^{1+\rho}$$

Now using $DELTA = \left(\frac{\delta}{1-\delta}\right)$ solve for δ as:

$$\delta = \frac{DELTA}{1+DELTA}$$

```
AC(I) = CC0(I) / ( BC(I)*M0(I)**(-RHOC(I))
          + (1-BC(I))*DC0(I)**(-RHOC(I)) )**(-1/RHOC(I) );
```

Do the same for the CET function

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BT(I) = 1/(1 + PD0(I)/PE0(I) * (E0(I)/DC0(I))**(RHOT(I)-1) ) ;
AT(I) = XS0(I)/( BT(I)*E0(I)**RHOT(I) + (1-BT(I))*DC0(I)**RHOT(I) )**(1/RHOT(I))
;

DISPLAY AC, BC, AT, BT ;

*##PRODUCTION FUNCTIONS: COBB-DOUGLAS TECHNOLOGY
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that is:

$$XS = AX \cdot K^{\alpha} L^{(1-\alpha)} \tag{9}$$

we called α as BX

from the foc of the producer, profşts are maximized when $MP_K = \pi$ which says that

$$\alpha = \frac{\pi K}{PX \cdot XS} \tag{10}$$

call πK as total profits = RP,

```
BX(I) = RP0(I)/(PX0(I)*XS0(I)) ;
AX(I) = XS0(I)/(K0(I)**BX(I)*LD0(I)**(1-BX(I))) ;
DISPLAY AX, BX ;
```