

Ec457 Alternative Theories of Growth and Distribution
Fall 2018, Final Exam

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1) (30 points)

Consider the R&D-driven endogenous growth model with the following technology for the production of the final good:

$$Y = L_Y^{1-\alpha} \sum_i^A x_i^\alpha$$

- a) Given the above technology, find the demand function for labor and the intermediate good, x_i .
- b) Suppose that one unit of x_i is being produced by foregone output. Thus the profit function of the intermediate good producer can be written as:

$$\pi(x) = p_i(x)x_i - rx_i$$

Find the optimal price in terms of a mark up and the marginal cost. What is the amount of the mark up? Find the optimal level of profits.

- c) Draw graphically the equilibrium of the oligopolist in the price-output space and interpret the optimal pricing rule of the oligopolist.
- d) Using the fact that aggregate “capital”, K , is $K = \sum x_i$, show that the above technology is equivalent to:

$$Y = K^\alpha (AL_Y)^{1-\alpha}$$

- f) Discuss the “no arbitrage” conditions in the finance and labour markets for this economy.
- g) Suppose that two countries, *Jonesia* and *Romeria* have the following research production functions, respectively:

Jonesia : $\dot{A}_J = \gamma L_J^2 A_J^{0.2}$

Romeria: $\dot{A}_R = \gamma L_R A_R^{0.9}$

With γ is the same across both countries, also suppose that both countries have the same population growth rate, $n=0.01$. Which country grows faster?

What can you say about the *real wage rates* in research in Jonesia versus Romeria? How do real wage rates change over time?

2) (20 points, 10 points each)

a) It is argued that for the R&D-driven endogenous growth models, the market solution usually leads to *pareto inferior* outcomes; *i.e.*, that growth and welfare can be improved upon the market solution by choice of an appropriate tax/subsidy scheme. Comment on this argument. What are the structural reasons of this particular trait?

b) Discuss the Schumpeterian concept of long run equilibrium based on the notion of *creative destruction*. How does the Romerian R&D function contrast with the Schumpeter's R&D function under creative destruction?

3) (20 points)

Consider the *Kaldorian* model where workers' saving rate is negative: -0.10, and investment is given from outside with $I = I^*$. There is no government spending and the economy is closed to foreign trade.

- a) Show the share of wages in output and verify that it is inversely related with the investment share of aggregate demand.
- b) What is the Keynesian multiplier for this economy? How is it related to the share of profit income in aggregate income?

4) (30 points)

Consider the following version of the Barro (1990) model of endogenous growth with a strategic public capital: Assume that per capita output, y , is given by the Cobb-Douglas function, with

$$y = f(k, g) = k^\alpha g^{1-\alpha}$$

where k is the (private) capital labor ratio and g is government spending per labor.

Labor is constant in supply; and there is no technological growth. The government finances its per capita expenditures by a lump-sum tax, T . Thus,

$$g = T$$

Under these conditions private capital per labor, k , accumulates given the law of motion:

$$\dot{k} = k^\alpha g^{(1-\alpha)} - c - T$$

Suppose that the instantaneous felicity of the representative consumer is given by the following:

$$U(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}$$

and the representative consumer maximizes the discounted utility,

$$\int_{t=0}^{\infty} e^{-\rho t} U(c_t) dt$$

Using a Hamiltonian function, set up this problem as in a *market setting* where the firms cannot internalize over the type of g and cannot affect its financing decision. Find the optimal growth of per capita consumption.